Angles and rotations

Here we will "store" the various equations

Quaternions

Avoid many problems computationally
conversions might be tricky

A quaternion is a hyper complex number.

$\hat{q} = q_i\, i + q_j\, j + q_k\, k + q_r$

with $i^2 = j^2 = k^2 = ijk = -1$

$\Rightarrow$ $ij = -ji = k$; $jk = -kj = i$; $ki = -ik = j$

if the rotation axis is $\hat{e} = \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix}$ and angle θ

the associated quaternion is expressed $\hat{q} = \begin{pmatrix} q_i \\ q_j \\ q_k \\ q_r \end{pmatrix} = \begin{pmatrix} e_x \sin(\theta/2) \\ e_y \sin(\theta/2) \\ e_z \sin(\theta/2) \\ \cos(\theta/2) \end{pmatrix}$

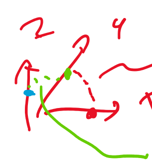
⚠ the other notation is common too

$q = q_r + q_i\, i + q_j\, j + q_k\, k$

$q = [q_r, q_i, q_j, q_k]$

And it is normalized $q_i^2 + q_j^2 + q_k^2 + q_r^2 = 1$

Rotation q followed by rotation $\tilde{q}$



R_1 $z(+90°)$
R_2 $x(+90°)$

R_1 then R_2
$= R_3$ $y(-90°)$

$$\tilde{q} \otimes q = \begin{bmatrix} q_r & q_k & -q_j & q_i \\ -q_k & q_r & q_i & q_j \\ q_j & -q_i & q_r & q_k \\ -q_i & -q_j & -q_k & q_r \end{bmatrix} \begin{bmatrix} \tilde{q}_i \\ \tilde{q}_j \\ \tilde{q}_k \\ \tilde{q}_r \end{bmatrix} = \underbrace{\begin{bmatrix} r & -k & j & i \\ k & r & -i & j \\ -j & i & r & k \\ -i & j & -k & r \end{bmatrix} \begin{bmatrix} i \\ j \\ k \\ r \end{bmatrix}}_{q \text{ ommitted for simplicity}}$$

Rotation Matrices

$$A_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \end{bmatrix}$$

$$A_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$A_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$A_z = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Euler 3-2-1 $\quad A = A_3 A_2 A_1 = A_z A_y A_x$

Multiply with a vector

compute $q^{-1} = \dfrac{q^*}{|q|^2}$

vector $\Rightarrow$ quaternion avec $\Re e$ Nulle

$v_{rot} = q^{-1} v_q q$

⚠ Rotation order.
One way to think about it is

$$\boxed{v_{rot} = \underbrace{q_b^{-1} \underbrace{q_a^{-1} v_q q_a}_{\text{first rotation}} q_b}_{\text{second rotation.}}}$$